An Investigation of Fatigue Failure of Materials under Broad Band Random Vibrations

T. C. HUANG*
University of Wisconsin-Madison, Madison, Wis.
AND
R. W. LANZ†
University of Wisconsin-Green Bay, Green Bay, Wis.

Theme

IN the present paper the fatigue life of material under the multifactor influence of broad band random excitations was investigated. Parameters which affect the fatigue life were postulated to be peak stress, variance of stress, and the natural frequency of the system. Experimental data were processed by the hybrid computer. Based on the experimental results and the regression analysis, a best predicting model has been found. All values of the experimental fatigue lives are within the 95% confidence intervals of the predicting equation. Research results in narrow band random fatigue have been reported.[1,2]

Contents

Parameters

When a material is under random excitations the time to failure depends on several parameters of the random responses. These parameters can be described in either the amplitude domain, the frequency domain, or in any combination of the two. In our tests the characteristics of the excitation as well as the response have a zero mean, and the amplitude distributions of the stress response are normal. Thus these two parameters, mean stress and stress amplitude distribution, are not in variation. The time to failure is then postulated to be related to the variance of stresses, the peak stress of the specimen and the natural frequency of the test system.

Equipment

The major piece of test equipment used was a 1200 lb MB-electromagnetic shaker with a sine-random signal generator. Recording instruments consisted of a Tektronics "Q" unit and an oscilloscope equipped with camera, and a Precision Instrument FM tape recorder. In addition, a GR spectrum analyzer was used.

The random output of the signal generator was fed into the MB-shaker amplifier. This produced a vertical motion of the shaker mass which strained the test specimen axially. The shaker mass, test specimen, load cell, and test fixture were connected in series. The load cell strain which was calibrated to the specimen strain was amplified by the "Q" unit, displayed on the oscilloscope and recorded by the FM tape recorder.

Specimen

The specimens, which are similar to tensile test specimens, were machined from a $\frac{1}{2}$-in.-diam 60-61 T651 aluminum rod. The diameter of the testing length of the specimen was 0.200 in. A large fillet with a radius of $\frac{3}{8}$ in. was used to minimize stress concentrations at the shoulders.

Test System

The natural frequency of the system was varied by two methods. The first was to use two different specimen test lengths, $3\frac{3}{16}$ in. and 4 in., which provided two different spring constants. The second method was to increase the natural frequency ranges by varying the shaker mass. This was done by bolting varying numbers of steel rings to it. Using these two methods, five different natural frequencies, 140, 170, 205, 250, and 300 cps, were used in the test.

The frequency input to the system was analyzed with the spectrum analyzer. The spectral density of the signal generator output is approximately constant up to 10,000 cps and then it rapidly falls off to zero at approximately 14,000 cps.

Test Procedure

The five desired peak stresses of the specimens in each of the five natural frequency groups ranged from 26,000 psi to 34,000 psi with 2000 psi increments. However, these exact values were difficult to obtain because of the random nature of the response. Because of the nature of the input random signal of the test equipment, the third factor, variance of stress was not directly controlled.

Approximately one-half way through the assumed specimen fatigue life, the "Q" unit output of the load cell strain was recorded on the tape for a length of 100 ft at 37.5 in./sec. When the specimen failed, the amount of time elapsed was recorded.

Data Analysis

The recorded portion of the load cell strain was then analyzed by the University of Wisconsin's hybrid computer. Figure 1 shows the response of a 140 cps system. The computer was programed to sample and store one million ordinates of the analog signal. An ordered array of voltage values, corresponding to the load cell strain, was printed. This showed the number of ordinates of the system response occurring within each 20 psi interval in the stress range of $\pm 60{,}000$ psi. The kurtosis and skewness of each response were calculated[3] from the array. This showed the amplitude distribution of this system to a white noise input was normal.

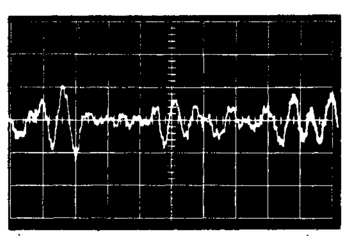

Fig. 1 140 cps system response.

Received August 5, 1971; synoptic received November 29, 1971; revision received February 7, 1972. Full paper is available from National Technical Information Service, Springfield, Va., 22151, as N72-17932 at the standard price (available upon request).

Index category: Properties of Materials.

* Professor, Department of Engineering Mechanics. Member AIAA.
† Assistant Professor, College of Environmental Sciences.

The peak stress was found in the array to be at the maximum address location in which at least one occurrence was stored. The computer was programed to calculate the variance of stress from the array by using standard statistical equations.

Mathematical Model

It was desirable to construct a mathematical model, based on experimental data, which would relate the fatigue failure time to the peak stress, the variance of stress and the natural frequency of the system. This was done by regression analysis.

A four-variable model represented by the following equation:

$$y = c_0 + c_1 x_1 + c_2 x_2 + c_3 x_3 \tag{1}$$

was tried in which y = function of fatigue time, x_1 = function of peak stress, x_2 = function of variance of stress, x_3 = function of the natural frequency of the systems, and c_0, c_1, c_2, and c_3 are constants.

Using standard correlation equations on various trial models the best model found is represented by

$$\log(T) = c_0 + c_1 \sigma + c_2 v + c_3 \log(f) \tag{2}$$

where T = fatigue time in sec, σ = peak stress in psi, v = variance of stress in (psi)^2, f = the natural frequency and the system in cps, and log represents $\log_{10}$. Let the series of tests performed be represented by

$$\bar{y} = X\bar{c} \tag{3}$$

in which $\bar{y}$ = column matrix of y_i, X = matrix of x, and $\bar{c}$ = column matrix of c_i. The column matrix $\bar{c}$ can be obtained as

$$\bar{c} = (X'X)^{-1}X'\bar{y} \tag{4}$$

in which X' = transpose of X and $(X'X)^{-1}$ = the inverse of $(X'X)$. Equation (4) was used to determine the coefficients c_0, c_1, c_2, and c_3 of Eq. (2) based on the data of 25 tests shown in Table 1.

Table 1 Test data, test results, and 95% confidence intervals

Specimen No.	System Natural Frequency cps	Peak Stress psi	Variance of Stress (psi)^{2}x10^{-7}	Observed Failure Time Sec.	Predicted Failure Time Sec.	95% Confidence Interval log(T)	
1	140	25,728	4.2210	42,240	20,485	3.9737	4.6760
2	140	27,794	4.9884	9,085	10,059	3.6532	4.3597
3	140	29,860	5.8091	6,343	4,773	3.3331	4.0333
4	140	30,026	6.7225	2,839	2,605	3.0590	3.7830
5	140	34,366	7.9936	956	728	2.5058	3.2305
6	170	26,451	4.3764	20,852	16,757	3.8783	4.5748
7	170	27,402	4.8748	5,080	11,001	3.7000	4.3881
8	170	30,481	6.2480	3,410	3,288	3.1801	3.8609
9	170	32,857	6.7123	1,250	1,899	2.9354	3.6294
10	170	35,895	8.1577	775	544	2.3795	3.1016
11	205	26,265	4.1059	21,052	19,858	3.9522	4.6456
12	205	27,980	4.6840	6,686	11,429	3.7185	4.4002
13	205	30,626	5.4536	3,970	5,274	3.3798	4.0678
14	205	32,237	6.5965	1,354	2,133	2.9926	3.6704
15	205	33,643	8.2498	430	635	2.4248	3.1878
16	250	26,286	4.5759	18,110	14,273	3.7939	4.5135
17	250	28,848	4.5938	11,460	10,780	3.6850	4.3802
18	250	31,617	5.3752	4,330	4,873	3.3287	4.0476
19	250	32,651	5.8075	3,412	3,309	3.1596	3.8811
20	250	33,374	7.0658	1,570	1,364	2.7891	3.4839
21	300	26,100	3.7183	30,630	24,700	4.0307	4.7514
22	300	26,079	4.5903	11,390	14,120	3.7694	4.5285
23	300	30,088	5.4489	7,028	5,331	3.3775	4.0751
24	300	33,023	6.1367	2,700	2,515	3.0424	3.7582
25	300	34,924	7.7050	1,090	750	2.5091	3.2433

Results

The results of this investigation are as follows:

1) Predicting equation. With the numerical values of c_0, c_1, c_2, and c_3 evaluated from Eq. (4), the fatigue predicting equation based on our tests applicable for the range of σ from 25,000 to 36,000 psi, is

$$\log(T) = 6.9401 - 4.5613 \times 10^{-5}\sigma - 2.7964 \times 10^{-8} v - 0.12798 \log(f) \tag{5}$$

Predicted values of fatigue life under broad band random excitations which are shown in Table 1 were calculated by using Eq. (5).

2) Confidence intervals. The 95% confidence intervals also shown in Table 1 were calculated by using standard statistical equations.[4] It was found that all the test results are within these intervals. However, it should be noted these intervals are large because there is only one observation for each test available for estimation.

3) Correlation coefficients. The correlation coefficients corresponding to Eq. (5) were calculated and the multiple correlation coefficient showed that 92% of the variation is explained by using all the variables. This percentage is greater than any other percentages obtained when not all variables are included. This means the model is best when all variables are included.

4) The response planes for peak stress, variance of stress, and frequency based on Eq. (5) are shown in Fig. 2. These planes give the over-all view of the fatigue life within the test ranges of the variables.

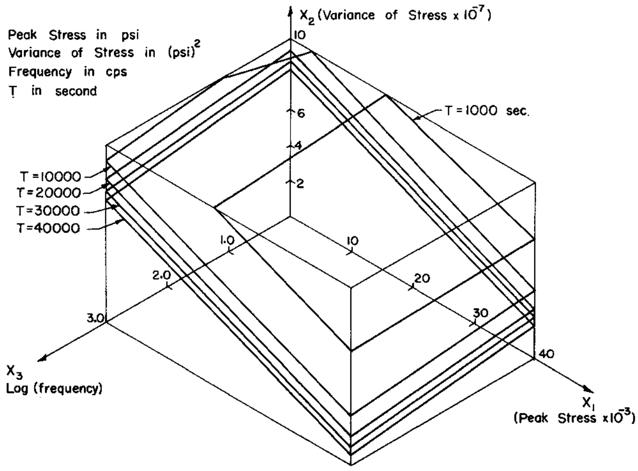

Fig. 2 Response planes.

Conclusion

In conclusion, a best model was found based on zero-order correlation coefficients. A further study of correlation coefficients indicates that all variables conjectured should be included. The fact that all experimental results are within the 95% intervals indicates that the model is good and experimental error is within the prescribed limits.

References

[1] Huang, T. C., Hubbard, R. B., and Lanz, R. W., "Fatigue Failure of Materials under Narrow Band Random Vibrations—Part 1," *Proceedings of the 21st International Astronautical Congress,* 1970, Constance, Germany, pp. 593–610.

[2] Huang, T. C., Hubbard, R. B. and Lanz, R. W., "Fatigue Failure of Materials under Narrow Band Random Vibrations—Part 2," *Proceedings of the 22nd International Astronautical Congress,* 1971, Brussels, Belgium, to be published.

[3] Croxton, F. E., Cowden, D. J., and Klein, S., *Applied General Statistics,* 3rd ed., Prentice Hall, Englewood Cliffs, N. J., 1967.

[4] Draper, N. R. and Smith, H., *Applied Regression Analysis,* Wiley, New York, 1966.